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DEFINITIONS OF THE TERM “MATHEMATICS.”

By DR. G. A. MILLER, University of Illinois.

As science advances many laws are found to be less simple than they were at first supposed to be and many terms acquire a breadth of meaning far beyond that assigned to them when they were first employed. In an age of such great scientific activity as the present, it might appear especially futile to try to define terms covering great fields of scientific inquiry, in view of the fact that each important new development tends to throw new light on the term under which it is classed. Moreover, as these developments proceed, the sciences tend more and more to grow into each other, and this complicates very seriously the formulating of satisfactory definitions. Broad definitions are not apt to meet with much opposition unless they encroach on the territory claimed by other terms by right of first exploration. As illustrative of this thought we need only point to the vague notions as regards the boundaries between arithmetic, algebra, and geometry.

Notwithstanding these serious difficulties, definitions of general terms frequently furnish most inspiring view-points of broad domains. Such a term as ‘‘Mathematics,’’ for instance, is continually becoming more replete with meaning for the growing mathematician, and, if he meets a definition which is just suited to his advancement, it tends not only to give him new pleasure but also to emphasize elements which had not been distinguished with sufficient clearness. Even if such a definition should not involve any element of novelty, it may give us pleasure by supporting a view which has been entertained by us for a long time. Probably few young mathematicians have read or re-read the remarks by Professor Bôcher on ‘‘Old and New Definitions of Mathematics,’’* without arriving at a clearer conception of some broad principle relating to mathematical thought.

The main object of the present note is a brief consideration of the statement: *Mathematics is the science of saving thought:*† In the first place, we may compare the scope of this statement with the well known definition which Benjamin Peirce gave in his Linear Associative Algebra (1870), viz.: *Mathematics is the science which draws necessary conclusions.* If, by saving thought, we mean the most economical use of thought these two definitions appear to have practically the same scope. For if we can draw a necessary conclusion in regard to a subject this conclusion may be drawn once for all by the same individual, and thereafter when circumstances arise which fulfil the conditions leading to the first conclusion the same result may be assumed without going through the various steps of reasoning. On the other hand, if we arrive at a definite conclusion without expending thought on all its details, it is essential that some of the intermediate steps were omitted in view of the fact that they could be reached by drawing necessary conclusions.

**Bulletin of the American Mathematical Society*, Vol. 11 (1904), p. 115.

†*Popular Science Monthly*, Vol. 73 (1908), p. 321.

Having observed that the definitions of the preceding paragraph have practically the same scope, the question arises whether this scope is not too comprehensive. It is easily seen that it includes developments which are not generally classed with Mathematics. It has been claimed that such developments are the result of the tendency of sciences to grow into each other and to encroach on domains which are not strictly their own. As is well known, some of the sciences which started out as non-mathematical, in their later developments availed themselves of mathematical results with great profit. A very striking instance is furnished by the ''Phase Rule'' of Professor Gibbs, which now occupies such a fundamental position in Physical Chemistry. Fairness, however, demands that the mathematician should not lay claim to territory first explored by other sciences even if it joins domains universally acceded to him and could have been properly claimed if the mathematician had been first to occupy it, or if the exploration had been accomplished by mathematical means.

In view of these facts it might be best to regard the so-called definitions under consideration as incomplete definitions, expressing concepts which dominate mathematical thought more completely than that of the other sciences. In his address before the recent International Mathematical Congress held at Rome, Poincaré developed the thought that *the role of science is to produce economy of thought,* which he attributed to Mach. He said that the savage calculates with his fingers or with little pebbles. In teaching children the multiplication table we save them innumerable manipulations with pebbles. In olden times someone recognized, with pebbles or otherwise, that 6 times 7 is 42 and he had the idea to note the result, and on this account we do not have to begin over again. This one did not lose his time even if he calculated only for pleasure; his operation took him only two minutes, while it would have required two billion minutes if a billion men would have been compelled to begin where he began. The importance of a fact is therefore measured by the quantity of thought which it enables us to economize.*

We desire to consider briefly another expression, viz., *Mathematics is a tool.* There is evidently some truth in this. A tool enables us to use physical energy more effectively just as Mathematics enables us to use thought with greater efficiency. On the other hand a tool is a simple instrument while Mathematics is very complex. It would be better to say that Mathematics consists of ten thousand tools, but this would not convey a complete idea, for Mathematics does not only develop a large number of simple tools but it especially emphasizes the putting together of these tools into powerful thought machines. The training in the construction of such

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**Bulletin des Sciences Mathematiques,* Vol. 32 (1908), p. 171.

machines is one of the greatest importance to those who expect to employ Mathematics intelligently in Physics or Engineering. If we are not mistaken Professor Slichter had practically the same thought in mind when he said: "It grates on me to hear Mathematics spoken of as a tool. Mathematics is to the engineer a basal science and not a tool. The spirit of that science is of more value to the engineer than the particular things that can be accomplished. The engineer need not be a mathematician, but he needs to think mathematically, and, to my mind, he needs the power of mathematical thought more than skill in manipulating a few mathematical tools in a mechanical fashion."*

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*Science, Vol. 28 (1908), p. 263.

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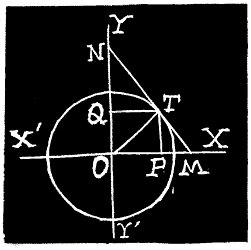
ON THE REPRESENTATION OF THE TRIGONOMETRIC FUNCTIONS BY LINES.

By R. D. CARMICHAEL, Alabama Presbyterian College.

The representation of the trigonometric functions dealt with in this note is in some respects different from that usually employed; it has certain advantages which appear to make it superior to other methods of effecting this representation.



Let the circle whose center is O (see figure) have the radius unity; and let O be the origin of the rectangular axes XX' and YY'. Let the angle $MOT=x$ be formed by the radius revolving counter-clockwise from OX to any position OT. Draw TP and TQ perpendicular, respectively, to OX and OY. At T draw a tangent to the circle cutting the X-axis in M and the Y-axis in N. Then in whatever quadrant OT may lie we have

$$\sin x=OQ,\quad \sec x=OM,\quad \tan x=TM,$$
$$\cos x=OP,\quad \csc x=ON,\quad \cot x=TN.$$

The tangent and the cotangent are measured from the point of tangency; the other functions from the center of the circle. Thus, all functions are measured from an extremity of the revolving radius.

It should be noticed that in any quadrant the tangent is that portion of the tangent line intercepted between the point of tangency and the X-axis, while the cotangent is intercepted between the point of tangency and the Y-axis. *This is the conception of Analytics.* The secant and the